Numeric Response Questions

Point

Q.1 A point P moves in such a way that sum of its perpendicular distanoes from two perpendicular lines in its plane is always 2. Then find the area of region bxunded by locus of P.

Q.2 Point Q is symmetric to $\mathrm{P}(4,-1)$ with respect to the bisector of the first quadrant. If length of PQ is $k\sqrt{2}$ then find k.

Q.3 If the extremities of the base of an isosceles triangle are the points $(2a, 0)$ and $(0, \mathrm{a})$ and the equation of one of the sides is $x = 2\mathrm{a}$. If the area of the triangle is ka^2 sq. units, then find k.

Q.4 If length of the median from B on AC, where $A(-1,3), B(1,-1), C(5,1)$ is $\sqrt{k}$ then find k.

Q.5. If the area of the triangle with vertices $(x, 0),(1,1)$ and $(0,2)$ is 4 sq unit, then find the value of x.

Q.6 If orthocentre and circumcentre of triangle are respectively $(1,1)$ and $(3,2)$ and the coordinates of its centroid are (h, k) then find $h+k$.

Q.7 Find the area of quadrilateral formed by the lines $3|x|+4|y|=6$.

Q.8 Find the area of triangle formed by the lines $xy=0$ and $x+y=4$.

Q.9 If (2, 3), $(-1,10)$ and $(4,5)$ are the vertices of a triangle then find the square of distance between circum center and orthocenter.

Q.10 The centroid of a triangle is $(1,4)$ and the coordinates of its two vertices are $(4,-3)$ and $(-9,7)$. Then find the area of the triangle.

Q.11 $\left(x_i, y_j\right)$ are vertices of a equilateral triangle ABC such that $\left(x_1-2\right)^2+\left(y_1-3\right)^2 = \left(x_2-2\right)^2+\left(y_2-3\right)^2=\left(x_3-2\right)^2+\left(y_3-3\right)^2$. Then $2\left(x_1+x_2+x_3\right)+3\left(y_1+y_2+y_{ij}\right)=$

Q.12 Opposite vertex of a square are $(-1,2)$ and $(3,4)$ then find the area of square.

Q.13 Mid-point of sides of a triangle in $x-y$ plane are $(1,1),(4,3)\&(3,5)$. Then find area of triangle.

Q.14 If the incentre of triangle formed by coordinates axes and line $3x+4y=12$ is (a, b) then find the value of $9a+8b$.

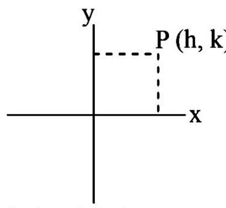
Q.15 If the vertices of a quadrilateral is given by $(x^2 - 9)^2 + (y^2 - 4)^2 = 0$ and area of quadrilateral is λ, then find the value of $\lambda/3$.

ANSWER KEY

1. 8.00	**2.** 5.00	**3.** 2.50	**4.** 10.00	**5.** –6.00	**6.** 4.00	**7.** 6.00
8. 8.00	**9.** 14.50	**10.** 91.50	**11.** 39.00	**12.** 10.00	**13.** 16.00	**14.** 17.00
15. 8.00						

Hints & Solutions

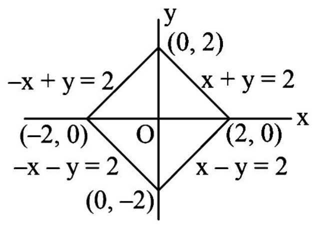
1. Let x axis and y axis be the two perpendicular lines. Let P be (h, k)



then | h | + | k | = 2

So locus of P is $\Rightarrow |x| + |y| = 2$

Which represents four lines forming a square.



$\text{Area} = 4\left(\frac{1}{2}\cdot 2\cdot 2\right) = 8$

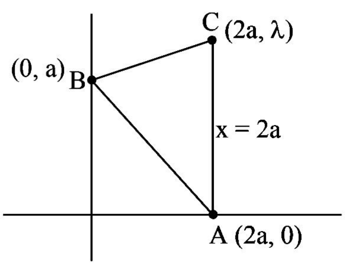
2. Bisector of the first quadrate

$\Rightarrow y = x$

Image of P(4, –1) w.r.t. to y = x is Q (–1, 4).

∴ Distance between the points P and Q is $5\sqrt{2}$

3.



$(CA)^2 = (CB)^2$

$\lambda^2 = 4a^2 + (\lambda - a^2)$

$\lambda^2 = 4a^2 + \lambda^2 + a^2 - 2a\lambda$

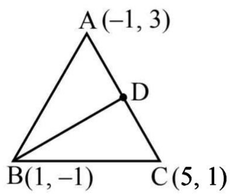
$2a\lambda = 5a^2 \qquad \Rightarrow \lambda = \frac{5}{2}a$

$$\Delta = \frac{1}{2}\begin{vmatrix} 2a & 2a & 0 & 2a \\ 0 & \frac{5a}{2} & a & 0 \end{vmatrix}$$

$$\Delta = \frac{1}{2}[5a^2 + 2a^2 - 2a^2] = \frac{5}{2}a^2$$

4. mid point of AC

D(2, 2)



$\text{Median BD} = \sqrt{(2-1)^2 + (2+1)^2} = \sqrt{10}$

5. Area of triangle = | 4 |

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x & 0 \\ 1 & 1 \\ 0 & 2 \\ x & 0 \end{vmatrix} = |4|$$

$\Rightarrow \frac{1}{2}[(x + 2) - (2x)] = |4|$

$\Rightarrow 2 - x = \pm 8$

$x = -6,\ x = 10$

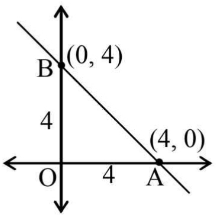
6.

2	1	
orthocentre (1, 1)	centroid	circumcentre (3, 2)

$\text{centroid}\left(\frac{6+1}{2+1}, \frac{4+1}{2+1}\right) = \left(\frac{7}{3}, \frac{5}{3}\right).$

7. $\text{Area} = \frac{2c^2}{ab} = \frac{2(36)}{3\times 4} = 6.$

8.



Area of $\Delta OAB = \frac{1}{2}(4)(4) = 8$

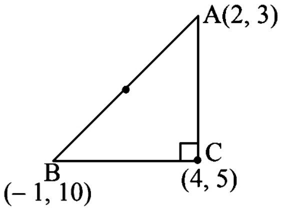
9. A(2, 3), B(– 1, 10) and C(4, 5)

$AB = \sqrt{(-1-2)^2 + (10-3)^2} = \sqrt{58}$

$BC = \sqrt{(4+1)^2 + (5-10)^2} = \sqrt{50}$

$CA = \sqrt{(4-2)^2 + (5-3)^2} = \sqrt{8}$

$\boxed{AB^2 = BC^2 + CA^2}$



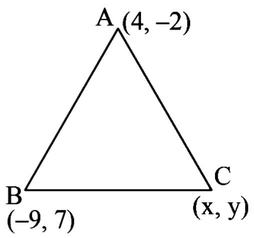
circum centre $\left(\frac{1}{2}, \frac{13}{2}\right)$

orthocenter (4, 5)

Distance $= \sqrt{\left(4-\frac{1}{2}\right)^2 + \left(5-\frac{13}{2}\right)^2}$

$= \sqrt{\frac{49}{4} + \frac{9}{4}} = \sqrt{\frac{29}{2}}$

10. centroid = (1, 4)



$\left(\frac{4-9+x}{3}, \frac{-3+7+y}{3}\right) = (1, 4)$

$\frac{-5+x}{3} = 1$ and $\frac{4+y}{3} = y$

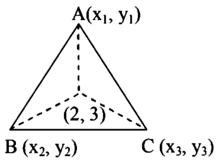
C (8, 8)

Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 4 & -3 \\ -9 & 7 \\ 8 & 8 \\ 4 & -3 \end{vmatrix}$

$= \frac{1}{2}$ [(28 – 72 – 24) – (27 + 56 + 32)]

$= \frac{183}{2}$

11.



As triangle is equilateral circumcentre and centroid coincide.

i.e. $x_1 + x_2 + x_3 = 6$

& $y_1 + y_2 + y_3 = 9$

Hence $2(x_1 + x_2 + x_3) + 3(y_1 + y_2 + y_3)$

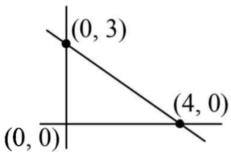
$= 2 \times 6 + 3 \times 9 = 39$

12. Area $= \frac{p^2}{2} = \frac{1}{2}[(3+1)^2 + (4-2)^2]$

$= \frac{1}{2}[16 + 4] = 10$

13. Reqd. Area $= 4 \times \frac{1}{2}\left\|\begin{matrix} 1 & 1 \\ 4 & 3 \\ 3 & 5 \\ 1 & 1 \end{matrix}\right\| = 16$

14.



So incentre is

$\left(\frac{5.0+4.0+3.4}{5+4+3}, \frac{5.0+4.3+3.0}{5+4+3}\right)$

$\equiv (1, 1)$

$\therefore$ a = 1, b = 1 $\Rightarrow$ 9a + 8b = 17c

15. $x^2 = 9 \Rightarrow x = \pm 3$

$y^2 = 4 \Rightarrow y = \pm 2$

hence vertices are (3, 2), (–3, 2), (–3, –2) & (3, – 2) respectively hence area of quadrilateral = 24